

GENETIC ALGORITHM IN CONCENTRATING EACH INDIVIDUAL GENETIC OPERATION ON A FUZZY SHORTEST PATH ALGORITHM

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ABSTRACT

The shortest path problem is an important classical network optimization problem arising from many applications including robotics, networking, VLSI design and transportation. In most situations, however, some issues of a network-theoretic problem may be uncertain. In conventional shortest path problems, there always an assumption that one who takes the decision is certain about the parameters (distance, time etc.) between different possible vertices in the network $G=\{V,E\}$. But while considering the real time cases, the possibility of existence of uncertainty about the parameters between different nodes is always high. In those situations, the representation of parameters are given by fuzzy numbers and here we consider the generalized trapezoidal fuzzy numbers, can be dealt with the uncertainty using fuzzy set theory. In order to provide solution for the uncertain shortest path problem, we proposed Genetic Algorithm (GA) in concentrating up gradation of each individual genetic operation. The proposed model is implemented using MATLAB with the test network of 30 nodes and the results reports that the algorithm converges in a more reasonable time in comparison with conventional approaches.

KEYWORDS: Genetic Algorithm, Generalized Trapezoidal Fuzzy Number, Selection, Population Initialization, Crossover, Mutation, Ranking Function, Shortest Path Problem

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1. INTRODUCTION

The Shortest path problem concentrates on finding the path with minimum distance. To find the shortest path from a source node to the other nodes is a fundamental matter in graph theory. In conventional shortest path problems it is assumed that the decision maker is certain about the parameters (distance, time, etc) between different nodes. But in real life situations, these always exists uncertainty about the parameters between different nodes. In such cases, the parameters are represented by fuzzy number and it was first analyzed by Dubois and Prade [3].

In order to make evolve the design of fuzzy systems, several met heuristic learning algorithms are projected. One major improvement class uses evolutionary algorithms (EAs) [9]. These algorithms are heuristic and random. They involve populations of individuals with a particular behavior like a biological development, like crossover or mutation [12]. The most well-known biological process fuzzy systems are the genetic fuzzy systems [2],[4]–[8],[10] that design fuzzy systems using Genetic Algorithms (GAs).

The view of the Genetic Algorithm (GA) should include concentration over various individual genetic operations. The various selection methods of the genetic algorithms are reviewed in [9],[11] which differ in the selection of individual

by two major criteria, one by the probability to the individual and other by the best individual. On the other hand, genetic operator crossover also have various successive methods is explained [6] andalso the genetically corrected process of obtaining chromosomes.

2. BASIC DEFINITIONS

The basic definitions of some of the required concepts are reviewed [4] in this section.

2.1 Fuzzy Set

Let X be an universal set of real numbers R , then a fuzzy set is defined as

$$A = \{[x, \mu_A(x)], x \in X\}$$

This is characterized by a membership function: $X \rightarrow [0, 1]$, Where, $\mu_A(x)$ denotes the degree of membership of the element x to the set A .

2.2 Characteristics of Generalized Fuzzy Number

A fuzzy set \tilde{A} which is defined on the universal of discourse R , is known to be generalized fuzzy number if its membership function has the following characteristics

- $\mu_A : R \rightarrow [0, 1]$ iscontinuous.
- $\mu_A(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
- $\mu_A(x)$ isstrictlyincreasingon $[a, b]$ andstrictlydecreasingon $[c, d]$.
- $\mu_A(x) = w$, for all $x \in [b, c]$, where $0 < w \leq 1$.

2.3 Membership Function of Generalized Trapezoidal Fuzzy Number

A generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ is known to be a generalized trapezoidal fuzzy number, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{w(x-d)}{(c-d)} & c \leq x \leq d \end{cases}$$

Let $\tilde{A} = (a, b, c, d; w)$ be a generalized trapezoidal fuzzy number then

$$\begin{aligned} \text{a) } R(\tilde{A}) &= \frac{w(a+b+c+d)}{4}, \quad \text{b) } M(\tilde{A}) = \frac{w(b+c)}{2}, \quad \text{c) divergence } D(\tilde{A}) = w(d-a), \quad \text{d) Left spread} \\ &LS(\tilde{A}) = w(b-a), \quad \text{e) Right spread } RS(\tilde{A}) = w(d-c) \end{aligned}$$

2.4 Fitness Function

The distance measure between the generalized trapezoidal fuzzy numbers \tilde{A} $(a_1, b_1, c_1, d_1; w_1)$ and \tilde{B} $(a_2, b_2, c_2, d_2; w_2)$ using centroid points (α, β) of \tilde{A} is given by [2]

$$f_d(\tilde{A}, \tilde{B}) = \max \{|\alpha_{\tilde{A}} - \alpha_{\tilde{B}}|, |\beta_{\tilde{A}} - \beta_{\tilde{B}}|, |R(\tilde{A}) - R(\tilde{B})|, |LS(\tilde{A}) - LS(\tilde{B})|, |RS(\tilde{A}) - RS(\tilde{B})|\}$$

$$\text{where } \alpha = \frac{1}{3} \left[a_1 + a_2 + a_3 + a_4 - \frac{a_4 a_3 - a_1 a_2}{(a_4 + a_3) - (a_1 + a_2)} \right] \text{ and } \beta = \frac{1}{3} \left[\frac{a_3 - a_2}{(a_4 + a_3) - (a_1 + a_2)} \right]$$

2.5 Addition of Fuzzy Numbers

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two trapezoidal fuzzy numbers then the addition is defined by $\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, w_1 + w_2)$

3. NETWORK TERMINOLOGY

Consider the directed network $G(V, E)$ consisting of a finite set of vertices $V = \{1, 2, \dots, n\}$ and a set of m directed edges $E \subseteq V \times V$. Each edge is denoted by an ordered pair (i, j) where $i, j \in V$ and $i \neq j$. In this network, we specify two vertices namely source vertex and the destination vertex. \tilde{d}_{ij} denotes the generalized trapezoidal fuzzy number associated with the edge (i, j) . The fuzzy distance along the path P is given in section 2.4.

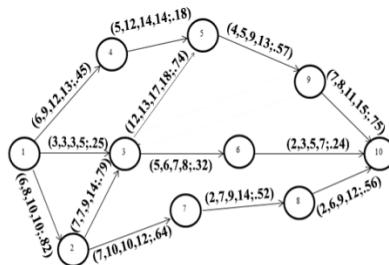


Figure 3.1

4. PROPOSED GENETIC ALGORITHM

Genetic Algorithm (GA) is a type of Evolutionary Algorithm (EA) which is based on the ‘natural selection’ phenomenon. GA usually has an analogy to the randomness in solving a problem. It is comprised of generations where children are produced by the mating of the parents with genetic operators. Selection and reproduction to produce efficient generation is based on the random procedures, known to be natural selection.

Our research is to scale the genetic algorithm in concentrating each and every genetic operator individually with a vision of increasing the convergence and providing standards for the randomness of ‘natural selection’ that leads to minimal the generations produced.

4.1 Representation of an Individual (Chromosome)

Each chromosome is represented in binary representation and it is also important which represents the solution in the generations. The representation defines the path traversed and indirectly refers the fuzzy fitness of the chromosome. The number of bits used in representing chromosome is equal to the number of vertices in the network graph $G = \{V, E\}$.

The vertex visited is represented by 1 and 0 represents that the vertex is not visited. Here, we take 10 vertices network and the representation 1101100001 represents that the path traversed may be 1-2-4-5-10, 1-2-5-4-10, 1-4-2-5-10, 1-4-5-2-10, 1-5-4-2-10 and 1-5-2-4-10 depending on the existence.

4.2 Population Initialization

The initial population is generated randomly in usual GA and each chromosome represents the collection of edges which are represented by generalized trapezoidal fuzzy numbers explained in previous sections. The default population size 20 is used.

Genetic Algorithm (GA) possesses random selection of chromosomes in initializing the population, in which chromosomes may uncertain in the existence as solution space. Randomness may also produce discontinuous path as the chromosome in initializing population and leads to unhealthy and unfitted generations. Since initialization of population implicates the convergence rate, the chromosomes should exist, continuous and provide optimal fitness in solution space.

Thus we propose a confined method for population initialization which produces healthy continuous valid chromosomes for the lead generation of the algorithm. The problem of avoiding discontinuous and duplicate or redundant chromosomes should be solved to meet our objective of healthy generation.

These problems can be solved by using adjacency matrix where 1 represents the availability of edge between vertices and in turns unavailability is represented by 0. The source and destination vertices are fixed and path is generated randomly.

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 $p = 1 \text{ to } Pop\_size$ 

 $src = start\_vertex, dest = end\_vertex, path[0] = src$ 

 $\text{for } i = 1 \text{ to } path[i - 1] != dest$ 

 $path_i = j, \text{where } j = \text{rand}\{\text{existing path from vertex } path_{i-1}\}$ 

 $Pop_p = path$ 

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The above pseudo code explains the method and possible existing path from vertex i can be determined by the adjacent matrix where columns representing 1 in i^{th} row are the vertices having edges from vertex i .

4.3 Selection Operation

Selection operation is used in initialization process and parent selection for crossover operation. Various selection operations involve Roulette wheel selection, Random selection, Rank selection, Tournament selection and Boltzmann selection [12].

Here we choose distance measure considering rank, divergence, left spread and right spread of generalized trapezoidal fuzzy numbers explained in previous section as the fitness function. As the fuzzy shortest path problem is considered, minimum will survive by the selection the chromosome values.

The probability to select the chromosome is individually calculated for all chromosomes in the generation and it is given by

$$P(\text{Pop}_p) = 1 - \frac{\text{fit}_p}{\text{fit}_{\text{total}}}$$

Where fit_p represents the fitness value of Pop_p and $\text{fit}_{\text{total}}$ represents the total fitness value. Since fitness value and probability are indirectly proportional, lesser the fitness gives greater the probability to select in the generation. Thus the best pair of chromosomes are selected to reproduce in each generations.

4.4 Crossover & Mutation Operation

Crossover operator mates two parent chromosomes and produces children which comprise the essence of two parent chromosome mated. Crossover operation is mainly categorized into two single point and multi point crossover

The single point crossover has single crossover site whereas multi point crossover has more than single crossover site. There are also some advanced multipoint crossover methods [12] and here we use a type of two point crossover technique.

The conventional mutation operator performs the minute changes of the reproduced child randomly under a certain rate which undo the degradation of the population due to crossover operation with crossover rate of 0.5.

There were many mutation operations for binary and real integers. Here we choose binary mutation that may be bit flipping, insertion, interchanging, reciprocal exchange, inversion and others [1].

The proposed method combines both the crossover and mutation in single operation. Since only the chromosomes having continuous path exists in the generation, we should also maintain its existence in crossover and mutation operation.

The parents having continuous path is connected through a mutator without affecting the existence (continuous) of the chromosomes. Bit flipping or addition is used as mutator in connecting the both parents to form next generations.

Consider an example with two parent chromosome A (1011001111) and B (1001101001). A two point crossover has to be carried out with a rate of 0.5 and mutation at 0.1.

Parent 1	Child
	1 – 2 – 7 – 8 – 10 1 – 2 – 4 – 5 – 9 – 10 (Crossover)
Parent 2	Child
	1 – 4 – 5 – 9 – 10 1 – 2 – 3 – 4 – 5 – 9 – 10 (Mutation)

Here, parents are merged with the connector (addition mutator) node 3 for obtaining continuous path. Bit flipping is also used to obtain continuous path.

4.5 Termination Condition

Termination condition produces the optimal solution through the convergence. Mostly termination condition will be the maximum number of generations. Other conditions are the idealness of the chromosomes in the generation. In order to test the algorithm, maximum number of generations can be used as termination condition which clearly represents the convergence of the algorithm.

Here, idealness of the chromosomes is considered as termination condition because of the usage trapezoidal fuzzy numbers and uncertainty in real numbers. When no change in the optimal fitness (minimal) and the idealness of the chromosomes in generations for at least 5 generations, then the algorithm reaches the termination condition. The network in section 3 after applying the proposed algorithm results the shortest path 1 – 3 – 6 – 10.

Algorithm

Step 1: Generate the network with vertices and edges with generalized trapezoidal fuzzy numbers.

Step 2: Population is initialized using proposed method explained in section 4.2

Step 3: Select the best chromosomes using proposed probability method in section 4.3

Step 4: Apply cross over process with the parent chromosomes selected using distance measure with the rate of 0.5 as explained in section 4.4.

Step 5: Mutation is carried out with a rate 0. 1 randomly as explained in section 4.4.

Step 6: when fitness of child is better than parent, child moves for next generation.

Step 7: Repeat the steps 3 to 5 till reaches the termination condition as explained in section 4.5.

Step 8: Report the chromosome with minimal fitness as the solution after termination condition.

5. NUMERICAL EXAMPLE

We Consider the network $G= \{V, E\}$ of vertices ($n=10$) represented in section 3. According to the assumption, we consider adjacent matrix for the finding of shortest path as initializing population of Genetic Algorithm (GA). Every edge is represented by the generalized trapezoidal fuzzy number. The fitness, ranking and properties of generalized trapezoidal fuzzy number in which described in previous chapters, are used to calculate.

Table 5.1: Sample Population Initialization

Path	Rand{Possible Vertex}	Selected Vertex
1	2,3,4	2
1-2	3,7	3
1-2-3	5	5
1-2-3-5	9	9
1-2-3-5-9	10	10
1-2-3-5-9-10	Reached Termination	Reached Termination

The population initialization is elaborately explained in our previous section 4.2. The distance measure will act as a fitness value of the chromosome $f_d(\tilde{A}, \tilde{B})$ given in section 2.4. Let consider two generalized trapezoidal fuzzy edges A(3,3,3,5;.25) and B(5,6,7,8;.32).

$$\alpha_A = 3.6667, \beta_A = 0.3334, R(A) = .875, LS(A) = 0, RS(A) = .5$$

$$\alpha_B = 6.5, \beta_B = .4167, R(B) = 2.08, LS(B) = .32, RS(B) = .32$$

$$f_d(\tilde{A}, \tilde{B}) = \max\{2.8333, .0833, 1.205, .32, .18\} = 2.8333$$

Table 5.2: Calculation $f_d()$ of Path 1-3-6-10

Path	Next Vertex	$f_d(\tilde{A}, \tilde{B})$ (Section 2.4)
1	3	0
1-3	6	0+2.833=2.833
1-3-6	10	2.833+2.214=5.047

Table 5.3: Fitness Values for Sample Paths

S. No	Path	$f_d(\tilde{A}, \tilde{B})$ (Section 2.4)
Pop_5	1-3-6-10	5.047
Pop_{10}	1-4-5-9-10	9.599
Pop_{15}	1-2-7-8-10	11.896
Pop_{20}	1-3-5-9-10	22.578

$$P(Pop_5) = 1 - \frac{fit_5}{fit_{total}} = 1 - \frac{5.047}{119.84} = 95.79$$

Table 5.4: Probability Values for Sample Paths

S. No	Path	$P(Pop_p)$ (Section 4.3)
Pop_5	1-3-6-10	.958
Pop_{10}	1-4-5-9-10	.920
Pop_{15}	1-2-7-8-10	.901
Pop_{20}	1-3-5-9-10	.812

Table 5.4 shows the probability values of sample paths and the paths Pop_5 and Pop_{10} having considerably greater probability are selected for the next generation. This continues till the termination is achieved and the shortest path obtained is 1-3-6-10.

6. IMPLEMENTATION & RESULTS

The implementation is carried out in Matlab 8.1 (R_{2013a}) 32 bit student version. The implementation is extended with our previous work and selection of valid path is controlled using adjacency matrix.

The network $G = \{V, E\}$ of 30 nodes with the edges of generalized trapezoidal fuzzy number is initialized. The algorithm is implemented as per the given description and demonstrated numerical calculation.

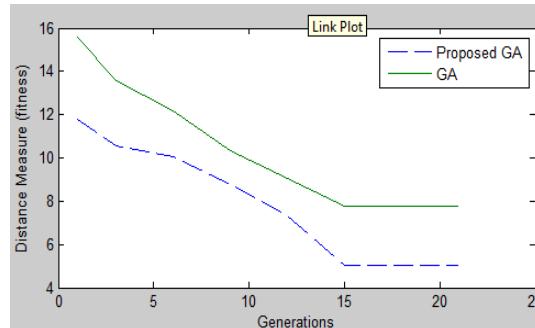


Figure 6.1: Comparison on Fitness Measures with Generations

From the figure 6.1, it is clear that the distance measure method depends on the rank, mode, divergence, left spread and right spread. The path at which all the components attain equilibrium is considered to be the shortest path. Here, generations around 15 – 21 in which chromosomes possess constant fitness and idealness in the distance measure generations and the path obtained is considered to be shortest path.

The huge difference in the start of the graph is due to the valid population initialization. The proposed crossover and mutation always produces the valid chromosomes with continuous path whereas conventional GA may fails with its crossover and mutation operation in initialization and further generations.

7. CONCLUSIONS

The Shortest Path (SP) problem in many applications is uncertain in parameters (Distance, Range, etc.). Hence, there occurs the necessity of fuzzy numbers for uncertain parameters. We propose a genetic algorithm in which concentrating each genetic operator individually upgraded to scale the genetic algorithm for greater convergence. The result clears that the proposed algorithm comprises the shortest path with minimal iterations.

The future enhancement of the research includes hybridization of the scaled genetic algorithm with the characteristics of ants which also keep track on the individual hybridization of genetic operators.

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